



## MODIFIED PHASE ACCEPTANCE IN LINACS

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December 10, 1970

### Purpose

To extend the calculation presented by Gluckstern<sup>1</sup> to obtain the phase width of the acceptance area of the synchronous energy.

### Calculation

Gluckstern<sup>1</sup> gives as the relation for the modified phase acceptance boundary:

$$y^2(x) = f(x) + \frac{5\epsilon}{2} \int_x^2 y(u) du \quad (1)$$

where

$$f(x) = (2 - x)^2 (1 + x). \quad (2)$$

His approximate analytical solutions do not cover the range  $x < -1$ .

To obtain these differentiate Eq. (1):

$$\left( 2 \frac{dy}{dx} + \frac{5}{2} \epsilon \right) y \cong f'(x). \quad (3)$$

Near  $y = 0$  and  $x = -1$  one expects that  $2dy/dx$  is quite large compared with  $5\epsilon/2$ . Hence, in this range

$$2y \frac{dy}{dx} \cong f'(x) \quad (4)$$

or

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$$y^2 = f(x) + \text{constant}. \quad (5)$$

The constant may be calculated using Gluckstern's solution for  $x = -1$ . Thus,

$$y^2(x) = y^2(-1) + f(x). \quad (6)$$

At the intercept  $y = 0$  one has

$$(2-x)^2 (1+x) = -y^2(-1) = -6 \epsilon. \quad (7)$$

To first order in  $\epsilon$  the left-hand intercept is

$$x = -1 - \frac{2\sqrt{3}}{3} \epsilon. \quad (8)$$

Gluckstern<sup>1</sup> gives  $x = 2$  for the right-hand intercept. Hence the phase width at the synchronous energy is

$$\Delta\phi = \left[ 2 - \left( -1 - \frac{2\sqrt{3}}{3} \epsilon \right) \right] \cdot \left| \phi_S \right|. \quad (9)$$

or

$$\Delta\phi = \left( 3 + \frac{2\sqrt{3}}{3} \epsilon \right) \cdot \left| \phi_S \right|. \quad (10)$$

### Results

Gluckstern<sup>1</sup> gives

$$\epsilon \approx 0.83 \sqrt{\frac{\Delta\beta}{\beta_S^2 |\phi_S|}}, \quad (11)$$

where  $\Delta\beta$  is the change in  $\beta$  per cell near injection. Using a PARMILA run, one finds

$$\begin{aligned} \Delta\beta &= 0.0015 \\ \beta_S^2 &= 0.0413 \\ \phi_S &= -32^\circ. \end{aligned} \quad (12)$$

Thus,

$$\epsilon = 0.21 \quad (13)$$

and

$$\Delta\phi = 103.76^\circ. \quad (14)$$

Correction to Account for Proper Initial Bucket Width

Gluckstern<sup>1</sup> approximates the initial bucket width as  $\Delta\phi = 3|\phi_S|$ .

To remove this approximation, one needs to employ the theory of Symon and Sessler.<sup>2</sup> For  $\phi_S = -32^\circ$  (Symon and Sessler use  $90^\circ - 32^\circ = 58^\circ$ ) the initial bucket width is

$$\Delta\phi = 122^\circ - 24.93^\circ = 97.07^\circ \quad (15)$$

which is to be compared with the approximate value used by Gluckstern.<sup>1</sup>

$$\Delta\phi = 3 \times 32^\circ = 96^\circ. \quad (16)$$

Hence, a correction of  $97.07 - 96^\circ = 1.07^\circ$  should be added to

Gluckstern's estimate of the bucket width. Thus, for  $\phi_S = -32^\circ$

$$\Delta\phi = 103.76 + 1.07 = 104.83^\circ. \quad (17)$$

## REFERENCES

- <sup>1</sup>R. L. Gluckstern, Phase Acceptance in Linacs, Proceedings  
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- <sup>2</sup>K. R. Symon and A. M. Sessler, Methods of Radio Frequency  
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